Int. J. Multiphase Flow Vol. 10, No. 1, pp. 61–77, 1984
Printed in Great Britain.

0301-9322/84 $3.00 + .00
Pergamon/Elsevier

A MODEL FOR THE FLOW OF SOLID PARTICLES IN GASES

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(*Received 7 January* 1983; *in revised form* 18 *March* 1983)

Abstract—The flow of solid particles in air streams involves a great deal of variables and complex phenomena, difficult to analyse. In practice the flow quantities in gas–solid flows are predicted by the use of empirical correlations of data or semi-empirical methods. The predictive power of these methods varies substantially between different systems. This paper presents an analytical approach to the subject of gas–solid flows, based on a turbulent model. The mixture is modeled as a variable density fluid flowing in a duct; the equations for the Reynolds stress incorporate the variation of velocity and density together, and yield the velocity profile of the flow and average quantities of interest such as the mass flux, the friction factor, the average density and average areas occupied by each phase. The predicted values for the friction factor are compared with known correlations emanating from experimental data. It is found that there is a very good agreement between the predicted values and the experimental correlations.

1. INTRODUCTION

The pneumatic transport of materials is an old and efficient technique for solid material transport, which is recently encouraged by the high cost of alternative transportation techniques. Pneumatic transport facilities are now widely used by industry for the transportation of a variety of solids through intermediate distances. The design of the components used is based primarily on experimental data and correlations. Since 1924, when the first scientific works appeared in the literature (Gasterstadt 1924; Cramp & Priestly 1924) there have been many experimental investigations reported in scientific journals on the subject of gas–solid flows. The theoretical analyses of the subject are still very few; in fact many of the analyses are simply using the separated flow model or dimensional analysis in order to develop correlations from the experimental data.

As regards the pressure drop in straight pipes the literature contains a number of experimental data sets and correlations, which are widely used by engineers: Rose & Duckworth (1969) made a dimensional analysis and developed a correlation based on their own pressure drop data. Their analysis includes ten dimensionless groups, some of which seem to have very small influence on the friction factor. Dogin & Lebeder (1962) developed a correlation by using only six groups; it seems that many experimental data (Konchesky *et al.* 1975a, 1975b) would agree fairly well with this correlation. Rose & Barnacle (1957) have used a correlation with only three dimensionless groups, while Pfeffer's *et al.* (1966) correlation only contains two groups, the loading of the flow and the Reynolds number of the gas phase, (from which the friction factor of the gas phase is derived). Jones *et al.* (1967) provide also a correlation for the frictional pressure drop, based on their own data. The last three correlations seem to use similar techniques and parameters with the pioneering work of Clark *et al.* (1953) who defined several of the variables now in use. During the seventies a great deal of experimental work was reported in three volumes of "Pneumotransport" (1971, 1973, 1980) emanating from an equal number of conferences organized by BHRA. Two monograms by Soo (1967) and Govier & Aziz (1972) and a handbook (Hetsroni 1982) and contain a great deal of information on the experimental work done in this field.

The only analytical approach to the subject appears to be Julian & Dukler's (1965) who developed an eddy viscosity model for the calculation of friction factors in gas–solid flows. This model assumes constant concentration profiles for the flows analyzed and employs an empirical correlation for the eddy viscosity of the fluid. However, the condition of constant concentration profiles imposes a severe restriction to the applicability of the model. Other investigators (Levy 1963; Maeder & Michaelides 1980; Shook & Daniel 1969) have developed similar models for gas–liquid or solid–liquid flows, using other restrictions, with various degrees of success.

The object of this work is to develop a model of two-phase gas–solid mixtures based on a variable-density turbulence set of equations. The flow will be taken to be in a circular duct. The fluid behaves as a single-phase fluid with variable time-averaged local density.The variations of the density are due to the fact that the distribution of solid particles in the pipe is not uniform. Experimental data by Soo *et al.* (1960, 1964) and Spencer *et al.* (1966) show that the particle concentration exhibits a maximum at the center of the pipe in the absence of electrostatic effects. The concentration profile for vertical pipes is symmetric and the distribution of the solid particles may be well approximated with a parabolic curve. Thus, the time-average local density of the flow will also exhibit a similar distribution. The variation of the local density as well as the velocity distribution are taken into account in the expression for the turbulent Reynolds' stresses. The shear stress expression yields the velocity profile, (which is different than that for a single-phase fluid with constant density); hence, certain average quantities are obtained, such as the mass flux, average velocity of the variable density fluid and space-average density. From the above, the friction factor is calculated as well as some average quantities of importance to the separated flow models, such as relative velocity, holdup and superficial velocities.

The model developed is based on phenomenological methods for the behavior of the variable density fluid. As such, it provides good results for the time-average flow quantities and resolves unknown conditions of local particle behavior. It is not, however, claimed to represent accurately the behavior of individual particles in the flow field or to answer questions about the interaction of particles. It must be considered as a mechanistic model that has good agreement with experimental results and predicts certain average quantities, useful to the designer of a pneumatic system.

2. FORMULATION OF THE PROBLEM

This study examines the adiabatic flows in a pipe carrying a symmetric suspension of solid particles in a gas. This situation may occur in practice in vertical pneumatic conveying systems. The objective of the study is to develop a model of the flow, which would predict the friction factor, pressure drop, and average quantities of interest such as mean velocities, discharge density and slip between the two phases.

The Navier–Stokes equations with Reynolds' stresses are developed. If the boundary conditions do not vary appreciably in one direction, then the problem posseses two length scales one very much larger than the other. Changes in fluid properties will occur more gradually in the longitudinal direction than in the transverse one. Thus, with the exception of pressure gradient all the other derivatives in the longitudinal direction can be neglected. The longitudinal changes in the fluid properties may be introduced later with no appreciable loss in accuracy. This technique has been tried successfully in other fields of fluid mechanics such as aerodynamics. Accordingly, the flow is assumed in one dimension, z, with $\partial u/\partial r \gg \partial u/\partial z$. The pressure gradient in the radial direction is assumed zero (a usual assumption in pipe flows) and the flow is assumed isothermal. The equation that governs the flow may be written in vectorial form:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \mathbf{g}\rho \qquad [1]$$

where

$$\mathbf{u} = \mathbf{u}(r), \qquad [2]$$

and

$$dp/dr = 0\,. \qquad [3]$$

Where **u** is the longitudinal velocity, ρ the density, p the pressure, μ the viscosity of the fluid, g the gravitational acceleration and r the radial distance. For a steady state flow, [1] yields:

$$\frac{dp}{dz} = \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{1}{r}\frac{d}{dr}(r\tau')\,, + g\rho\,, \qquad [4]$$

in which τ' is the Reynolds stress, μ the viscosity and ρ the density of the fluid.

3. THE REYNOLDS STRESS

In a compressible flow field it has been shown by Schlichting (1978) and Pai (1957) that:

$$\tau' = -\overline{(u+u')(v+v')(\rho+\rho')}\,, \qquad [5]$$

where u and v are the time-average values of the velocity vector in the longitudinal and radial direction, the primes denote the turbulent perturbations and the bar time-average. Bearing in mind that in a one-dimensional flow $v = 0$, [5] yields:

$$\tau' = -(\rho\,\overline{u'v'} + u\,\overline{\rho'v'} + \overline{u'\rho'v'})\,. \qquad [5a]$$

The last term may be neglected as being the product of three small fluctuation terms and thus, of one order of magnitude lower than the first two terms. Therefore, the expression for the Reynolds stress becomes:

$$\tau' = -(\rho\,\overline{u'v'} + u\,\overline{\rho'v'})\,. \qquad [5b]$$

It is apparent that the Reynolds stress is comprised of two momentum exchange terms one stemming from velocity fluctuations and the other from the combined effect of density and velocity fluctuations. The last term is always zero in studies pertinent to incompressible single-phase flows.

The choice of closure relationship for the Reynolds stress τ' must satisfy the dissipation inequality dictated by the second law of thermodynamics:

$$\tau'\frac{du}{dy} \geq 0\,, \qquad [6]$$

which states that the Reynolds stress must act in the direction of the velocity gradient.

Therefore, the shear stress of the fluid at any radial position is the sum of the viscous and the turbulent components:

$$\tau = \mu\frac{du}{dy} + \tau'\,, \qquad [7]$$

where

$$y = r_0 - r\,, \qquad [8]$$

is the distance from the wall of the pipe and r_0 the pipe radius.

It is usual to divide the fluid region of the pipe in two parts: the viscous sublayer, extending from the wall to a distance y_0, and the turbulent region from y_0 to the center of the pipe. The viscous stress $\mu(du/dy)$ is the dominant term in [8] inside the viscous sublayer, while the Reynolds stress τ' dominates in the turbulent region. Hence, we may write:

$$\tau \approx \mu \frac{du}{dy}, \quad 0 \le y < y_0; \tag{9}$$

and

$$\tau \approx \tau' \quad y_0 \le y \le r_0 . \tag{10}$$

The shear stress τ has the value zero at the center of the pipe and its maximum value τ_w is attained at the wall.

For a steady-state flow situation the momentum equation reduces to a force balance equation which reads:

$$\frac{r}{2}\frac{dp}{dz} + \tau + \frac{g}{r}\int_0^r \rho r \, dr = 0 . \tag{11}$$

This equation yields for the shear stress τ_w at the wall:

$$\frac{r_0}{2}\frac{dp}{dz} + \tau_w + \frac{g}{r_0}\int_0^{r_0} \rho r \, dr = 0 . \tag{12}$$

When the pressure gradient dp/dz is eliminated between the above two equations the expression for the shear stress becomes:

$$\tau = \frac{r}{r_0}\tau_w + g\left[\frac{r}{r_0^2}\int_0^{r_0} \rho r \, dr - \frac{1}{r}\int_0^r \rho r \, dr\right], \tag{13}$$

where the density ρ is a function of r as confirmed by experimental results (Soo 1960). The functional form of ρ will be specified later.

Since the width of viscous sublayer y_0 is very small in comparison with the pipe radius r_0 it is often assumed that the apparent stress at y_0 is equal to the wall shear stress τ_w (Schlichting 1978)

$$\tau_w \approx \tau(y_0) = \tau'(y_0) = \rho_G V^{*2} . \tag{14}$$

The quantity V^* is the shear velocity of the flow and is defined in terms of [14]. The shear velocity is a fictitious velocity related to the velocity gradient rather than the average velocity itself.

It is necessary to use a closure equation for τ' in terms of the velocity gradient, since the form of [5b] is not suitable for computations given that the fluctuations ρ', u' and v' are unknown. This closure equation is subject to the constraint of [6] for positive entropy production. There are several hypotheses in single phase flow that yield a suitable form for the closure equation. Among them are Prandtl's mixing length theory (1925), von Karman's similarity hypothesis (1950) and Taylor's vorticity hypothesis (1932). All of them lead to similar results for the velocity distribution in the pipe and consequently agree well on the friction factors for the single-phase flows.

Here, a technique similar to the mixing-length theory will be followed and the shear stress τ' will be given in the following form:

$$\tau' = \rho l_u^2 \left|\frac{du}{dy}\right|\frac{du}{dy} + u l_u l_\rho \left|\frac{d\rho}{dy}\right|\frac{du}{dy}. \qquad [15]$$

The arguments that lead to the formulation of [15] are similar to the ones used by Prandtl (1925) and Pai (1957) and take into account the density fluxuations. The mixing lengths l_u and l_ρ must be zero at the wall and must be related to some other length of the problem. Here Prandtl's suggestion is followed, for simplicity, that these lengths are proportional to y, which is the only natural dimension of the problem:

$$l_u = l_\rho = \kappa y. \qquad [16]$$

This expression has been preferred over more complicated ones, which, nevertheless give similar velocity profiles for single-phase flow.

A change of variables in [15] will yield the following expression in terms of the boundary layer coordinates:

$$\tau' = \kappa^2\left[\rho\left|\frac{du}{d\eta}\right| + u\left|\frac{d\rho}{d\eta}\right|\right]\frac{du}{d\eta}, \qquad [17]$$

where η is the boundary layer coordinate:

$$\eta = \ln(y/y_0). \qquad [18]$$

As explained above, y_0 is the length of the laminar (viscous) sublayer and its value may be calculated from the following equation, which is valid for single-phase flow: (see Appendix A for its derivation):

$$y_0 = 0.111\, k(\nu_G/kV^* + 0.3). \qquad [19]$$

where k is the roughness of the pipe and ν_G the kinematic viscosity (μ_G/ρ_G) of the gaseous phase. It may be seen that if $\nu_G/kV^* \ll 0.3$ the roughness effects predominate and the pipe is characterized as rough, while in the opposite situation the pipe is smooth. For the integration with the boundary layer coordinate η the ratio r_0/y_0 is needed to define the upper limit of integration η^*:

$$\eta^* = (\ln(r_0/y_0) = \ln(r_0/k) - \ln(\nu_G/kV^* + 0.3) + 2.198. \qquad [20]$$

Thus far, the closure equation for the Reynolds stresses has been derived in terms of natural or boundary layer coordinates. A specification of the density distribution in the flow is still needed in order to derive a closed form relationship for the stress τ_0.

4. THE DISTRIBUTION OF DENSITY

This study examines flows where the heavier phase (solid) is carried by a gaseous matrix, which is continuous. Experiments by Soo *et al.* (1960, 1964), Spencer (1966) and Peskin (1967), indicate that at the absence of electrostatic forces the solid will concentrate towards the center of the pipe. Therefore, at the wall of the pipe the average density is very close to the gas density. At the center of the pipe the average density attains its highest value, ρ_m which is nevertheless lower than the density of the solid phase. The experimental results for the density distribution show that the functional form of the density distribution depends on the total solids concentration. Correlations of experimental data indicated that a suitable form of the density distribution function is:

$$\rho = \rho_G(1 + \gamma y/r_0)^m, \qquad [21]$$

where γ and m depend on the total solid concentration. Then the maximum density ρ_m at the center of the pipe is:

$$\rho_m = \rho_G(1 + \gamma)^m. \qquad [22]$$

It appears in the works of Soo *et al.* (1960, 1964), Spencer (1966) and Peskins (1967) that m is a weak function of the total solids concentration and varies between 0.4 and 0.6. Therefore, m could be assumed to be a constant and attribute the whole density variation to γ. For the calculation described later m is taken to be 0.5 and γ can vary from 0 (single phase fluid) to $(\rho_s/\rho_G)^{1/m} - 1$ (if at the center of the pipe only solid particles flow). Actually, the latter situation does not occur in practice, since there is always some aire between any two solid particles. Therefore the above value for γ was considered as an upper limit for the density distribution and was not used in the calculations.

Given this form of density distribution some practical quantities of interest may be calculated. The space-average density $\bar{\rho}$ is given by the following expression:

$$\bar{\rho} = \frac{1}{\pi r_0^2}\int_0^{r_0} 2\rho\pi r\,\mathrm{d}r. \qquad [23]$$

After substituting $y = r_0 - r$ and the value of ρ from [21] the above expression yields:

$$\frac{\bar{\rho}}{\rho_G} = \frac{2}{\gamma^2(m+1)(m+2)}[(1+\gamma)^{m+2} - 1]. \qquad [24]$$

The area-average density of the fluid is related to the average area occupied the solids $\bar{\alpha}$ (equivalent to the average void fraction in gas–liquid flows) as follows:

$$\bar{\rho} = (1 - \bar{\alpha})\rho_G + \bar{\alpha}\rho_s. \qquad [25]$$

Thus, $\bar{\alpha}$ may be determined from the density distribution function. Another quantity, the local area fraction $\alpha(r)$ is also important for calculations. It is given by the following equation:

$$\rho = (1 - \alpha)\rho_G + \alpha\rho_s, \qquad [26]$$

where $\rho = \rho(r)$ has given by [21] and $\alpha = \alpha(r)$. It is evident that if [26] is space averaged, it will yield [25]. The local solids area fraction is a function of the radial position of the pipe and not a constant as other investigators have assumed (Julian & Dukler 1965). This fact is supported by the experiments mentioned above.

5. THE VELOCITY PROFILE

In view of the function of the density distribution, the shear stress distribution may be written in the following form:

$$\tau' = \frac{r}{r_0}\tau_w + \rho_G r_0 g\left[xG(1) - \frac{1}{x}G(x)\right] = \kappa^2 y^2\left[\rho\left|\frac{\mathrm{d}u}{\mathrm{d}y}\right| + u\left|\frac{\mathrm{d}\rho}{\mathrm{d}y}\right|\right]\frac{\mathrm{d}u}{\mathrm{d}y}. \qquad [27]$$

The above emanates from [13] and the function $G(x)$ is defined by the following integral:

$$G(x) = \int_0^x \rho/\rho_G x \, \mathrm{d}x, \qquad [28]$$

with $x = r/r_0$, and $y = r_0(1-x)$.

After substituting in [27] for the density distribution and the wall shear in terms of the shear velocity ($\tau_w = \rho_G V^{*2}$) one may obtain:

$$\frac{r_0 - y}{r_0}\rho_G V^{*2} + \rho_G r_0 g\left[xG(1) - \frac{1}{x}G(x)\right] = \kappa^2 y^2 \left[\rho_G\left(1 + \frac{r}{r_0}y\right)^m \left|\frac{\mathrm{d}u}{\mathrm{d}y}\right| + u\left|m\rho_G\frac{\gamma}{r_0}\left(1+\gamma\frac{y}{r_0}\right)^{m-1}\right|\right]\frac{\mathrm{d}u}{\mathrm{d}y}. \qquad [29]$$

Equation [29] may yield the velocity distribution in terms of the shear velocity V^*. In order to assist in the calculations, the gravity term in brackets can be evaluated explicitly and the resulting expression can be written in terms of x:

$$H(x) = xG(1) - \frac{1}{x}G(x) = \frac{1}{\gamma(m+1)}[(1+\gamma-\gamma x)^{m+1} - x] + \frac{1}{\gamma^2(m+1)(m+2)}\left[\frac{1}{x}(1+\gamma-\gamma x)^{m+2} - x + x(1+\gamma)^{m+2} - \frac{1}{x}(1+\gamma)^{m+2}\right]. \qquad [30]$$

A dimensionless velocity $u^* = u/V^*$ is defined and also a Frounde number, F^*, based on the shear velocity.

$$F^* = \frac{V^*}{\sqrt{(gr_0)}}. \qquad [31]$$

F^*, hence, is the ratio of the viscous to the gravity forces. In view of the above the shear stress of [17] may be written in dimensionless form and in terms of the boundary layer coordinate η:

$$(1 - \mathrm{e}^{\eta-\eta_0}) + \frac{1}{F^{*2}}H(\eta) = \kappa^2\left[\rho^*\left|\frac{\mathrm{d}u^*}{\mathrm{d}\eta}\right| + u^*\left|\frac{\mathrm{d}\rho^*}{\mathrm{d}\eta}\right|\right]\frac{\mathrm{d}u^*}{\mathrm{d}\eta}; \qquad [32]$$

where the function H is written in terms of η:

$$H(x) = H(1 - \mathrm{e}^{\eta-\eta_0}), \qquad [33]$$

as defined explicitly in [30]. Here,

$$\rho^* = \rho^*(\eta) = (1 + \gamma\,\mathrm{e}^{\eta-\eta_0})^m. \qquad [34]$$

The spatial coordinates x and y are written in terms of η as follows:

$$x = 1 - \mathrm{e}^{\eta-\eta_0}, \qquad [35]$$

and

$$y = r_0\,\mathrm{e}^{\eta-\eta_0}. \qquad [36]$$

The velocity profile $u^*(\eta)$ may be actually obtained from [32], which is a quadratic equation with respect to the velocity gradient $du^*/d\eta$.

$$\frac{du^*}{d\eta} = \frac{-B + \sqrt{(B^2 - 4AC)}}{2A} \qquad [37]$$

where

$$A = \kappa^2 \rho^*, \qquad [37a]$$

$$B = \kappa^2 u^* |d\rho^*/d\eta|, \qquad [37b]$$

and

$$C = -1 + e^{\eta - \eta_0} - \frac{1}{F^{*2}} H(\eta). \qquad [37c]$$

Given that $C < 0$ and $A > 0$ [37] has always one positive root which is an acceptable value for $du^*/d\eta$. Actually in this case where the density increases along η the absolute values in [32] are not necessary to be written explicitly, and the terms multiplying $du^*/d\eta$ in brackets may be written simply as $d(u^*\rho^*)/d\eta$. The case would not have been the same if the solid particles had less density than the carrier fluid. Then the absolute value terms in the density profile should have been retained further and the terms in brackets could not have been written as above.

One glance at [37] shows that it is a first order nonlinear ordinary differential equation with the boundary condition $u^*(-\infty) = 0$ (i.e. at the wall of the pipe the velocity is zero). Based on what was said above about the boundary layer coordinates and the choice of the laminar sublayer, this condition may be approximated to:

$$u^*(0) = 0. \qquad [38]$$

Now [37] may be solved numerically to yield the velocity distribution $u^*(\eta)$. This task is accomplished numerically and the results are shown in figure 1 for the following conditions: $m = 0.5$, $\eta_0 = 8$ and $\kappa = 0.4$; the loading $\dot{m}^*$ is a parameter. It is observed that the depicted velocity profile differs from the incompressible flow prifile, which is given by the simpler expression

$$u^* = K\eta + L. \qquad [39]$$

This difference is due to the presence of the density gradient terms, which introduce an extra term in the force balance equation. Julian & Dukler (1965) obtained the same type of velocity profiles by determining empirically the eddy diffusivity of the flows.

6. AVERAGE VELOCITY AND MASS FLUX

In the case of pipe flow the average velocity $\bar{u}$ (or equivalently the volumetric flux per unit area) and the mass flux G are two quantities of interest to the engineer. These are defined as follows in terms of the spatial coordinate r:

$$\bar{u} = \frac{2}{r_0^2} \int_0^{r_0} ur \, dr = V^* I_1, \qquad [40]$$

and

$$G = \frac{2}{r_0^2} \int_0^{r_0} u\rho r \, dr = V^* \rho_G I_2. \qquad [41]$$

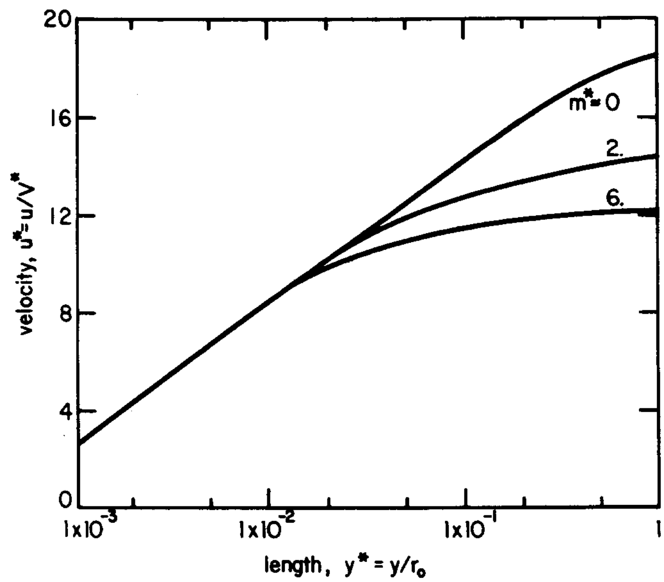

Figure 1. Velocity distributions for various loadings.

In the above two expressions u and ρ must be given in terms of the spatial variable x. The integrals I_1 and I_2 are dimensionless and are representative of the above average quantities.

Using integration by part the two integrals may be written as:

$$I_1 = -\frac{1}{r_0^2}\int_0^{r_0} \frac{du^*}{dr} r^2\, dr, \qquad [42]$$

and

$$I_2 = -\frac{1}{r_0^2}\int_0^{r_0} \frac{d(u^*\rho^*)}{dr} r^2\, dr. \qquad [42a]$$

Substituting into boundary layer coordinates we may express the integrals by using [32] and [37].

$$I_1 = \int_0^{\eta_0} \frac{du^*}{d\eta}(1 - e^{\eta-\eta_0})^2\, d\eta \qquad [43]$$

and

$$I_2 = \int_0^{\eta_0} \frac{d(u^*\rho^*)}{d\eta}(1 - e^{\eta-\eta_0})^2\, d\eta = \int_0^{\eta_0} \frac{(1 - e^{\eta-\eta_0}) + \frac{1}{F^{*2}}H(\eta)}{\kappa^2 \frac{du^*}{d\eta}}(1 - e^{\eta-\eta_0})^2\, d\eta. \qquad [44]$$

As explained before the final form of integral I_2 may be written as in [44] only because both u^* and ρ^* are increasing functions of η. Otherwise, the derivative $d(u^*\rho^*)/d\eta$ will have to be evaluated and I_2 would be calculated from [44]; this would involve double integration.

It is common engineering practice (Govier & Aziz 1972) to express the average quantities in a complex mixture in terms of the superficial velocities V_s^s and V_G^s, defined as follows:

$$V_s^s = \frac{\dot{m}_s}{\rho_s A}, \qquad [44]$$

and

$$V_G^s = \frac{\dot{m}_G}{\rho_G A}, \qquad [45]$$

where $\dot{m}$ is the mass flow rate and A the total area. The superficial velocity of each phase is the fictitious velocity, that this phase would have if it was flowing alone in the pipe.

The average velocity $\bar{u}$ in the pipe is:

$$\bar{u} = V_s^s + V_G^s = I_1 F^* \sqrt{(g r_0)}, \qquad [46]$$

and the average mass flux G:

$$G = \rho_s V_s^s + \rho_G V_G^s = I_2 F^* \rho_G \sqrt{(g r_0)}. \qquad [47]$$

The actual average velocity of each phase may be obtained from the following equations:

$$\bar{u}_G = \frac{V_G^s}{1-\alpha} \qquad [48a]$$

and

$$\bar{u}_s = \frac{V_s^s}{\alpha}. \qquad [48b]$$

It is apparent that integrals I_1 and I_2 would yield easily all the above average quantities.

Another quantity of interest to the engineer is the discharge density, ρ_d:

$$\rho_d = G/\bar{u}, \qquad [49a]$$

which is in general different than the average density $\bar{\rho}$ because of the existence of slip. The discharge density may be written in terms of the two integrals I_1 and I_2 as follows:

$$\rho_d = \rho_G I_2 / I_1. \qquad [49b]$$

7. THE FRICTION FACTOR

One may define a friction factor f for the pipe according to the Fanning equation:

$$\frac{\mathrm{d}p_f}{\mathrm{d}z} = \frac{2 f \rho_d \bar{u}^2}{D}, \qquad [50]$$

where $\mathrm{d}p_f/\mathrm{d}z$ is the frictional component of the pressure loss, and D the diameter of the pipe.

In terms of the integrals I_1 and I_2 the above equation may be written:

$$\frac{\mathrm{d}p_f}{\mathrm{d}z} = \frac{2 f \rho_G V^{*2} I_1 I_2}{D}. \qquad [51a]$$

The shear stress τ_w at the wall is:

$$\tau_w = \frac{1}{4}\frac{\mathrm{d}p_f}{\mathrm{d}z} D = \frac{1}{2} f \rho_G V^{*2} I_1 I_2. \qquad [51b]$$

Thus, given that $\tau_w = \rho_G V^{*2}$ we may write the friction factor f as follows:

$$f = \frac{2}{I_1 I_2}. \qquad [52]$$

The last equation combined with [51] and [31] yields:

$$\frac{\mathrm{d}p_f}{\mathrm{d}z} = \frac{4}{D}\rho_G g r_0 F^{*2} = 2g\rho_G F^{*2}. \qquad [53]$$

Therefore, the value of the Froude number F^* of the flow will yield the frictional pressure gradient and in turn the shear stress at the wall. Hence, one only has to determine F^* for a given situation in order to obtain the shear stress at the wall. This task is cumbersome and would involve the solution of the integral equation [41] (if the mass flux is known) or [40] (if the average velocity is known). The solution of either equation can be achieved by iteration. In practice one would usually know the "loading" m^* of the flow (which is the ratio $\dot{m}_s/\dot{m}_G$) and the volumetric flow rate of the gas ($\dot{Q}_G = \dot{m}_G/\rho_G$). Then the mass flux of the system is:

$$G = \dot{Q}_G \rho_G / A(1 + m^*), \qquad [54]$$

and [41] becomes an integral equation with F^* as the only unknown:

$$\frac{\dot{Q}_G \rho_G}{A}(1 + m^*) = F^* \rho_G \sqrt{(g r_0)} I_2. \qquad [41a]$$

It has been a practice in the past (Rose & Duckworth 1969; Konchesky 1975a, 1975b; Pfeffer *et al.* 1966) to give a frictional pressure drop coefficient f_m based on the gas-phase flow variables. This coefficient is defined as follows:

$$\tau_w = \frac{1}{2} f_m \rho_G (V_G^{s2}), \qquad [55]$$

where V_G^s is the superficial velocity of the gas; f_m is the fictitious coefficient what would yield the same shear τ_w with the gas–solid system when multiplied with the kinetic energy of the gas phase when flowing alone in the pipe. In the literature f_m is often given as a function of m^* and Re_G^s (Hetsroni 1982). Given that $V_G^s = \dot{Q}_G/A$ the practical importance of f_m is obvious: One can treat the solid–gas flow as a single-phase flow, obtain the term V_G^s and calculate τ_w easily after f_m has been estimated from correlations or tables.

8. COMPARISON OF THE RESULTS WITH CORRELATIONS AND OTHER EXPERIMENTAL DATA

Equations [51b] and [55] may be combined to yield an expression of f_m in terms of the Fanning coefficient f:

$$f_m = f\left(\frac{V^*}{V_G^s}\right)^2 I_1 I_2 = 2\left(\frac{F^{*2} r_0 g}{V_G^{s2}}\right). \qquad [56]$$

Thus, the friction factor of the mixture f_m can be calculated from the flow quantities. This friction factor is plotted in figure 2 vs the Reynolds number Re_G^s of the flow based on the superficial gas velocity:

$$\mathrm{Re}_G^s = \frac{V_G^s \rho_G D}{\mu_G}. \qquad [57]$$

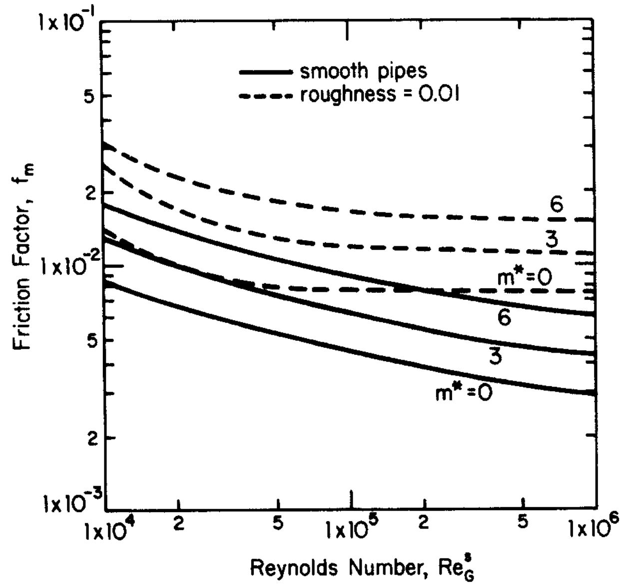

Figure 2. The friction factor for different loadings and pipe roughnesses.

The plot is for two loadings $m^* = 2.5$ and $m^* = 6.0$ and also for a smooth pipe ($k/r_0 = 0$) and for a rough pipe ($k/r_0 = 0.01$). As expected higher loadings or increased roughness yield a higher friction factor. It is often argued that the passage of abrasive solids through the pipes will smoothen them and hence that most of the available experimental data correspond to smooth-pipe data. This trend was also observed with comparisons made with the present model; it seems though that most of the data compared exhibit friction factors slightly higher than those predicted for smooth pipes, thus indicating a small amount of residual pipe roughness. In the comparisons that follow the smooth-pipe results of this model will be compared to the other methods of prediction of friction factors.

The results obtained by the present model were also compared with correlations of experimental data. First the correlation by Pfeffer *et al.* (1966) was compared and the results are depicted in figure 3. The Pfeffer correlation was developed from collected sets of data and it

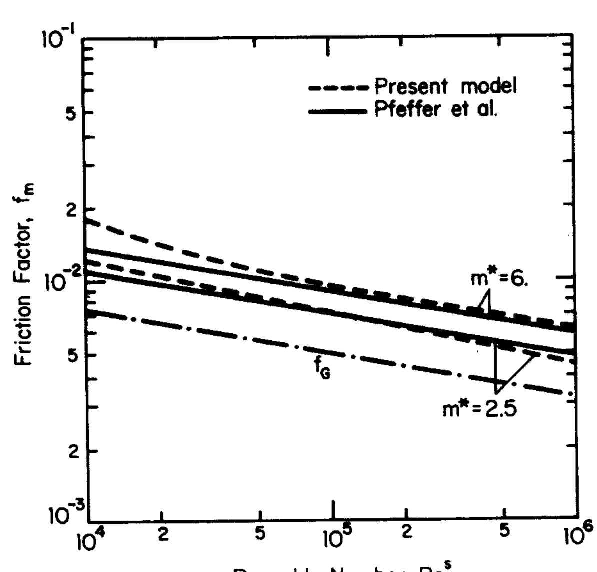

Figure 3. Comparison of the results from present model with Pfeffer's correlation.

was said to predict lower values for f_m (Hetsroni 1982) below $Re_G^s = 10^5$. This trend is observed in figure 3 also since Pfeffer curves lie even below the smooth pipe curves for this model. However, the discrepancy is very small and it is within the standard error of the correlation.

Figure 4 shows the results obtained from this model together with results obtained from other correlations used for the predictions of the friction factor in gas–solid flows. The correlations by Dogin & Lebedev (1962), Rose & Barnacle (1957) and Pfeffer *et al.* (1966) are shown for $m^* = 6$ and the ratio of densities $\rho_s/\rho_G = 1000$. It is apparent that there is good agreement between the results from this study and other correlations. Again, the present results would have been higher (and would be closer to the other experimental curves) if a finite roughness for the pipe was chosen.

Finally, the results of this study are compared to those obtained by Julian & Dukler (1965). They used the data from six experimental studies (Hariu & Molstad 1949; Mitlin 1954; Helander 1956; DePew 1960; Clark *et al.* 1952; Hinkle, 1953) in order to correlate the eddy diffusivity of the gas–solid mixtures. Their model assumed constant density profile and their velocity profiles are similar to the ones shown in figure 1. The comparison of the Julian and Dukler curves with those predicted from the present model are shown in figure 5, for $m^* = 1$, 3 and 5. It is observed that there is good agreement between the corresponding curves. The good agreement of the end result for the two models indicates that the observed higher eddy diffusivity in the past papers is entirelly due to the density variation across the pipe. This phenomenon is taken into account in [27] by the inclusion of the term $u|d\rho/dy|$ for the contribution of the density to the Reynolds stress. Thus, there was no reason to use an empirical relation for the eddy diffusivity.

As pertains to the average densities predicted by this model figure 6 depicts a plot of ρ/ρ_G vs the loading ratio m^*. The solid line represents the approximate equation.

$$\bar{\rho}/\rho_G \approx 1 + m^*, \quad [58]$$

as verified experimentally and as used by Hetsroni (1982) and Govier & Azis (1972), and the circles results from this model. It is obvious that there is almost no discrepency between the predicted and the correlation values. On the same graph the flow area $\bar{\alpha}$ occupied by the solids is shown as derived from the average density results.

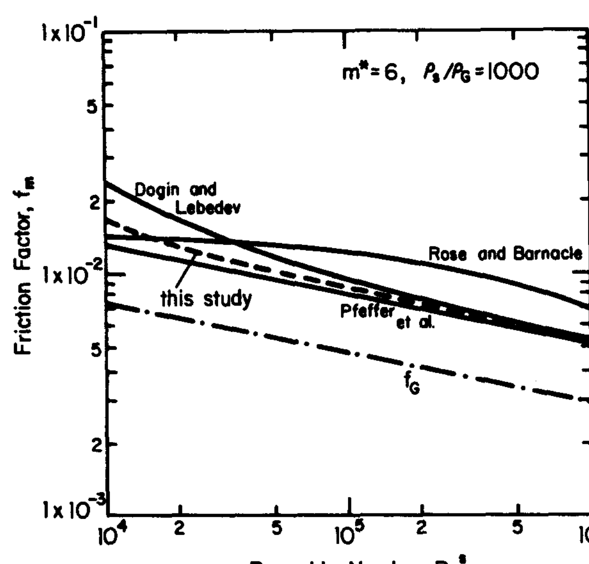

Figure 4. Comparison of the results from the present model with results emanating from empirical correlations.

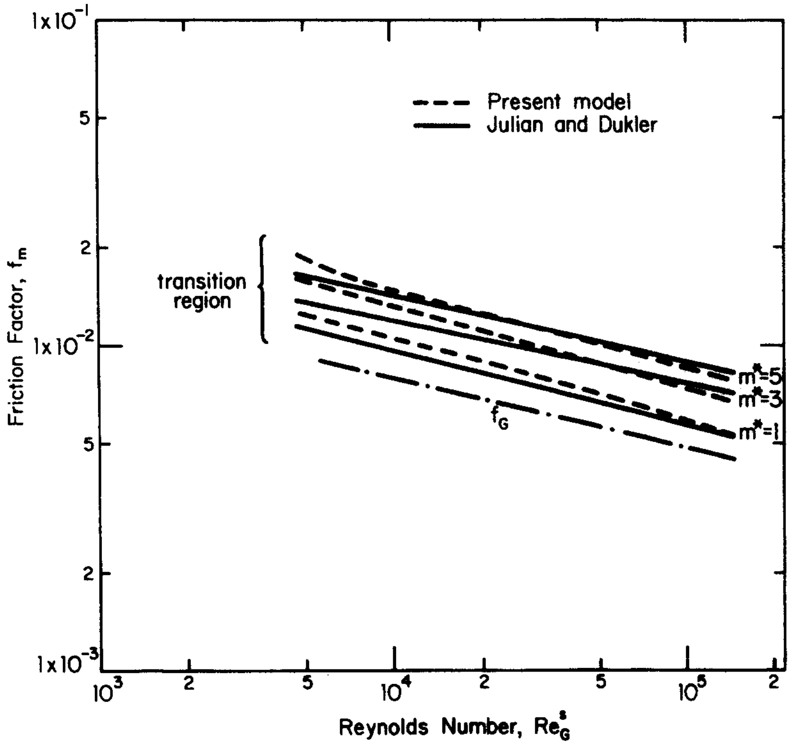

Figure 5. Comparison of the results from the present model with the ones from Julian and Dukler's model.

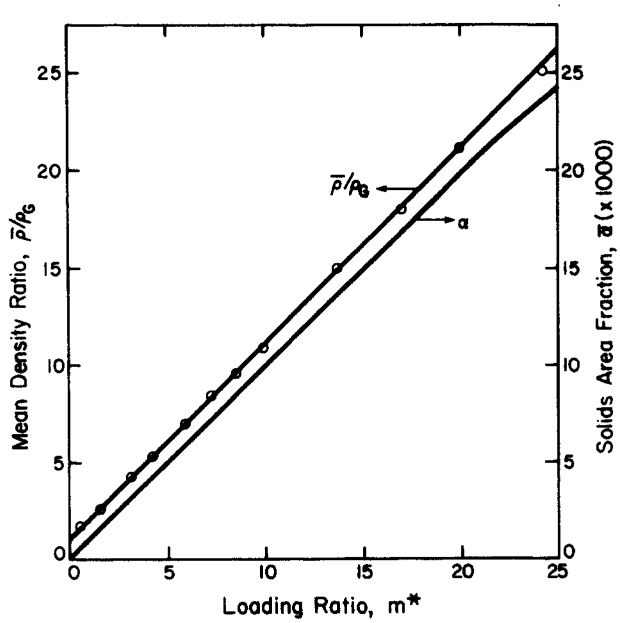

Figure 6. The average density $\bar{\rho}$ and solids area fraction $\bar{\alpha}$ vs. the loading ratio $m^*(\rho_s/\rho_G = 1000)$. The circles denote results of the present model.

The preceding equations were developed for vertical pipe flows. However, the gravity terms $[xG(1) - G(x)/x]$ in the equation for the shear stress [27] are very small in comparison to the term $\tau_w r/r_0$. Even if these terms are neglected there is very little difference in the friction factor, f_m. Therefore, the results obtained here may be used for hirozontal pipes, provided that the density distribution in them is symmetric (fine or medium size particles in high velocity flows).

It can be seen in figure 4 that there is a discrepancy in the correlations for the friction factor. This discrepancy is due to the fact trhat certain parameters, such as the size, shape and distribution of particles are not taken under consideration in many correlations.

It must be emphasized that the present model is a mechanistic one and attempts to describe the gas–solid flows regardless of some details of the system, such as the shape and size of particles or the spread of the sizes of particles. It describes the time-average flow and predicts certain flow parameters for the complex mixtures. The general trend and the magnitudes of the variables for the flows predicted agree very well with experimental data. However, there is not very accurate prediction of certain sets of data in flows where sizes and shapes of particles play an important role. This is by no means refuting the validity of the model, since even the best regarded correlations also do not agree with many sets of experimental data (Martin & Michaelides 1983). The subject of air–solid flows is a very complex one and involves a great deal of variables. One may improve on the model by including the effects of all the important variables. It is thought that a more complex turbulent model, such as the κ–ϵ model, would improve the prediction of the present one by including the effects of particle size and shape. The detailed discussion of such a model is beyond the scope of this paper. At the moment it seems that the results of this study are sufficiently accurate despite the simple approach to the modeling of two-phase systems.

9. CONCLUSIONS

A model was developed for gas–solid two-phase pipe flows based on a "zero equation" turbulence model. The complex mixture was taken to be a homogeneous fluid of variable density across the pipe cross-section. The Navier–Stokes equations for such a fluid were solved for the steady-state flow. The eddy diffusivity of such a system is enhanced by the variation of the density. The Reynolds' stresses for the flow exhibit two principal terms, which are calculated according to the mixing length hypothesis.

The resulting model may be used to predict the superficial velocities of the two phases, the holdup and slip of the flow, as well as the frictional pressure drop. Also the average densities, mass flux and average velocities of the mixture may be easily calculated. The results of the calculations show a very good agreement with experimental data and existing correlations. Certain parameters in this study (such as the mixing lengths and the diffusivity terms) were determined arbitrarily because of the lack of experimental data. Extensive experimentation is needed for the determination of these parameters. Also needed are more data on the density distribution parameters, which here were determined according to a limited number of known data.

Acknowledgements—This work was inspired by previous work of the author with Professors Maeder and DiPippo of Brown University, on the subject of vapor–liquid flows. The help of both through many discussions is greatly appreciated. The work was partly supported by a grant from the university of Delaware Research Foundation.

REFERENCES

Clark, R. H., Charles, D. E., Richardson, J. F. & Newitt, D. M. 1952 Pneumatic conveying—I. The pressure drop during horizontal conveyance. *Trans. Inst. Chem. Engng* **30**, 209–224.

Cramp, W. & Priestly, A. 1925 Pneumatic grain conveyors. *Engineer* **137** 34, 64, 89, 112.

De Pew, C. A. 1960 Heat Transfer to flowing gas–solid mixtures in a vertical duct. Ph.D. Thesis, University of California.

Dogin, M. E. & Lebedev, V. P. 1962 Dependence of the resistance in pneumatic conveying pipelines on the fundamental parameter of the two-phase flow. *Ind. Engng Chem.* **2**, 64–68.

GASTERSTADT, J. 1924 Die Experimentalle Unterschung des Pneumatischen fosterorgays. Forsch. Arb. *Ingeneur Wes.* **68**, 265–280.

GOVIER, G. W. & AZIZ, K. 1977 *The Flow of Complex Mixtures in Pipes.* Krieger, Huntington, New York.

HARIU, O. H. & MOLSTAD, M. C. 1949 Pressure drop in vertical tubes in transport of solids by gases. *Ind. Engng Chem.* **40**, 1148–1160.

HELANDER, R. E. 1956 The flow of air–solid mixtures. Ph.D. Thesis, Northwestern Univ., Ill.

HETSRONI, G. (Ed.) 1982 *Handbook of Multiphase Systems.* McGraw–Hill, New York.

HINKLE, B. L. 1953 Acceleration of particles and pressure drops encountered in horizontal pneumatic conveying. Ph.D. Thesis, Georgia Inst. Tech.

JONES, J. H., BROWN, W. G., DAUBERT, T. T. & ALLENDORF, H. D. 1967 Estimation of pressure drop for vertical pneumatic transport of solids. *AIChE J.* **13**, 608–613.

JULIAN, F. M. & DULKER, A. E. 1965 An eddy viscosity model for friction in gas–solids flow. *AIChE J.* **11**(5), 853–858.

VON KARMAN, T. 1950 Mechanishe Ahnlingkeit & Turbulenz. Nachr Ges. Wiss. Gottingen, Math. Phys. Klasse 58 (1930). Also coll. works II, 337–346.

KONCHESKY, J. L., GEORGE, T. J. & CRAIG, J. G. 1975a Air and power requirements for the pneumatic transport of crushed coal in horizontal pipelines. *Trans. ASME J. Engng Ind.* **97**, 94–100.

KONCHESKY, J. L., GEORGE, T. J. & CRAIG, J. G. 1975b Air and Power requirements for the pneumatic transport of crushed coal in vertical pipes. *Trans. ASME J. Engng Ind.* **97**, 101–106.

LEVY, S. 1963 Prediction of two phase pressure drop and density distribution from mixing length theory. *Trans. ASME J. Heat Transfer* **85**, 137–152.

MAEDER, P. F. & MICHAELIDES, E. E. 1980 Prediction of the entropy production and pressure losses in two phase flow from mixing length theory. Brown Univ. Rep. No. GEO-FLO/2DOE/ET/27225-11.

MARTIN, J. & MICHAELIDES, E. E. 1983 A critical review of frictional pressure-drop correlations for gas–solid flows. *Proc. 3rd Multiphase Flow and Heat Transfer Symp.–Workshop*, Miami.

MITLIN, L. 1954 Air and solid velocities in pneumatic conveying. Ph.D. Thesis, London Univ.

PAI, S. I. 1957 *Viscous Flow Theory Chapter II. Turbulent Flow.* Van Nostrand, Princeton.

PESKIN, R. L. 1966 Fluid mechanics and transport properties of gas–solids suspensions. U.S. Dept. of Interior, Bureau of Mines.

PFEFFER, R., ROSSETTI, & LICKLEIN, S. 1966 Analysis and correlation of heat transfer coefficient and heat transfer data for dilute gas–solid suspensions. NASA TN–D 3603.

Pneumotransport 1. 1971 First international conference on the pneumatic transport of solids in pipes. BHRA.

Pneumotransport 2. BHRA, 1973.

Pneumotransport 5. BHRA, 1980.

PRANDTL, L. 1925 Uber die ausgebildete Turbulenz. *ZAMM* **5**, 136–139.

ROSE, H. E. & BARNACLE, H. E. 1957 Flow of suspensions of non-cohesive spherical particles in pipes. *Engineer* 898–901, 939–941.

ROSE, H. E. & DUCKWORTH, R. A. 1969 Transport of solid particles in liquids and gases. *Engineer* 392–396, 430–433, 478–483.

SCHLICHTING, H. 1978 *Boundary layer theory* (Translated by J. Kestin), 6th Edn. McGraw–Hill, New York.

SHOOK, C. A. & DANIEL, S. M. 1969 A variable density model of the pipeline flow of suspensions. *Can. J. Chem. Engng* **47**, 196–201.

SOO, S. L. 1967 *Fluid Dynamics of Multiphase Systems.* Blaisdell, Waltham.

SOO, S. L. & REGALBUTO, J. A. 1960 Concentration distribution in two-phase pipe flow. *Can. J. Chem. Engng* **38**, 160–165.

SOO, S. L., TREZEK, G. J., DIMIC, R. C. & HONSTREITER, G. F. 1964 Concentration and mass flow distribution of a gas-solid suspension. *Ind. Engng Chem. Fund.* **3**, 98–105.

SPENCER, J. D., JOYCE T. J. & FABER, J. H. 1966 Pneumatic transport of solids. U.S. Dept. of Interior, Bureau of Mines Circ. 8314, 62–72.

TAYLOR, G. I. 1932 The transport of vorticity and heat through fluids in turbulent motion. *Proc. Roy. Soc. London* **A135**, 685–705.

VOGT, E. G. & WHITE, R. R. 1948 Friction in the flow of suspensions; granular solids gas-through pipes. *Ind. Engng Chem.* **40**, 1731–1738.

APPENDIX A

The velocity distribution in the laminar layer is linear with respect to y, while in the turbulent region it is given as

$$u^* = 2.5 \ln \frac{V^* y}{\nu_G} + 5.5\,, \qquad \text{[A1]}$$

for smooth pipes, or

$$u^* = 2.5 \ln \frac{y}{k} + 8.5\,, \qquad \text{[A2]}$$

for rough pipes.

As explained in section 5 the end of the laminar sublayer is taken as the projection of the u^* line in the u^*, $\ln y$ plane where u^* becomes 0. This is manifested in [38] as $u^*(y = y_0) = 0$. Accordingly the above equations yield:

$$y_0 = 0.111\,\nu_G/V^*. \qquad \text{[A3]}$$

for smooth pipes or

$$y_0 = 0.0334\,k\,, \qquad \text{[A4]}$$

for rough pipes.

The combination of the last two equations yields [19]. It is observed that this choice of y_0 reduces the width of the laminar sublayer and this makes the assumptions $u^*(y = y_0) = 0$ and $t(y_0) = \tau_w$ more realistic. It must be emphasized that the final results are not sensitive to the value of y_0. Other constants were tried and found to yield similar results.